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Testing Predictive Developmental Hypotheses

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Predictive developmental hypotheses play a crucial role in developmental theories. These hypotheses link early developmental behaviors or processes to later developmental outcomes. Empirical tests of predictive developmental hypotheses are generally based on standard regression models. It is argued that hierarchical linear models or longitudinal multilevel models offer a better alternative. A multivariate longitudinal model linking developmental data to a criterion is described and an application is given. The application, derived from attachment theory, pertains to the prediction of infant behavior in the Strange Situation. It is concluded that the proposed approach offers a valuable tool to the developmentalist, both from a theoretical and methodological point of view.

Many developmental psychological hypotheses derive their attractiveness from the fact that they predict behaviors and attributes later in life. Predictive developmental hypotheses typically link behaviors and processes in an early phase of development to later outcomes.

According to Freud's psychoanalytic theory for instance, the foundation of adult personality is laid very early in life. If the child's psychosexual development is arrested during the second part of the fist year of life, the adult personality will be characterized by a quest for knowledge and power (Miller, 1993). Freud (1941) used verbal accounts of clients and colleagues to justify his claims. Of course his methods are seen as inadequate today.

The sensitivity attachment hypothesis of Ainsworth et al. (1978) offers another example of a predictive developmental hypothesis. According to attachment theory (Bowlby, 1969, Ainsworth, 1978) the mother's early sensitivity affects the child's quality of attachment later in life. When a mother is sensitive to her infant's signals and needs, and responds to them appropriately, the infant learns to trust his/her mother and a secure mother-infant attachment relationship results. Ainsworth and many that followed her used correlational analyses to test the sensitivity attachment hypothesis (Goldsmith & Alansky, 1987). In a recent meta analysis, de Wolf and van IJzendoorn (1997) report a mean correlation of r = .24 between sensitive

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responsiveness in the first year of life and the quality of attachment in the second year of life.

Good developmental theories lead to developmental predictions. In general, theories that predict individual outcomes are preferred above theories that only predict group means (Thomas, 1996). Statistical models that allow for *individual* predictions are therefore crucial to developmental research. Hierarchical linear models or multilevel models can be very helpful in this respect.

Bryk and Raudenbush (1987, 1992), Francis et al.(1991), Goldstein (1986, 1995), Plewis (this issue) and others showed that the hierarchical linear model or multilevel model is well suited to analyze longitudinal data. Hoeksma and Koomen (1992) showed that the longitudinal multilevel model fits in well with the methodology of developmental research, as it was put forward by Wohlwill (1973) and Baltes and Nesselroade (1979). In this article we will show how hierarchical linear models or multilevel models can be used to investigate predictive developmental hypotheses.

The model to be described has its roots in the study of growth and development. The use of polynomials, which play an important part in the longitudinal multilevel model, dates back to Wishart (1938). He used polynomial functions to describe weight-growth of pigs. By now classic work on the comparison of polynomial coefficients across groups was done by Box (1950), Rao (1959, 1965) and Grizzle and Allen (1969). Goldstein (1986) showed how physical growth can be modeled efficiently from a multilevel perspective. More recently latent growth models have become an important tool in the study of growth and development (e.g. McCardle & Epstein, 1987; Muthén, 1991; Willet & Sayer, 1994).

The multilevel models to be described derive to a large extent from Goldstein (1989). He showed how the longitudinal multilevel model can be used to predict adult physical height (c.f. Goldstein, 1995).

Longitudinal Predictions

A predictive developmental hypothesis is a hypothesis that links early developmental processes or behaviors to later developmental outcomes. Empirical tests of developmental hypotheses have to be based on longitudinal data (Baltes & Nesselroade, 1979). In actual studies the developmental process will be charted by means of repeated measures of one or several variables. The outcome, criterion, or developmental status to be predicted will be measured later in time. The interval spanned by the hypothesis may range from just a few days to many years.

There are at least two approaches to test developmental hypotheses. The first approach uses the regression model to predict the developmental outcome. The second approach is based on dynamic models. In the latter case extrapolation is used. The curve describing the early developmental changes is extended to the age of the developmental outcome to be predicted. Both approaches will be discussed briefly. Subsequently we will show how the longitudinal hierarchical linear model or multilevel model combines the strengths of both approaches.

The following situation will be considered. A sample of N individuals is observed at $T + 1$ (possibly fixed) occasions. The observations pertinent to the early developmental process are designated by y_{ti}, where $t = 1,...,T$ refers to the measurement occasion, and $i = 1,...,N$ refers to the person. The variable a_{ti} refers to the age of person i at occasion t. The outcome of the developmental process or criterion to be predicted for person i at occasion $T + 1$ is designated by y_{ci}.

To illustrate our reasoning a hypothetical example will be used. It is hypothesized that the *development* of a person's vocabulary during early childhood *predicts* the size of his or her adult vocabulary. The hypothetical sample consists of N individuals, with repeated measurements of vocabulary size (y_{ti}) at yearly intervals from $a_{1i} = 1$ to $a_{5i} = 5$ years. Adult vocabulary (y_{ci}) is measured at 21 years of age.

Regression Approach

In many instances predictive developmental hypotheses are tested by means of regression analysis. The analysis consists of finding a linear combination of the developmental variables y_{ti} that optimally predicts the developmental outcome y_{ci}. The well-known model is:

$$(1) \qquad y_{ci} = \lambda_0 + \sum_{t=1}^{T} \lambda_t y_{ti} + e_i \text{ with } e_i \sim N(0,\sigma_e^2),$$

where y_{ci} is the observed measurement of the developmental outcome of individual i at occasion $T + 1$ (e.g. adult vocabulary size) and y_{ti} refers to the observation of a developmental process of person i at occasion t (e.g. repeated measurements of vocabulary size during childhood). The parameters to be estimated are the intercept λ_0 and the regression weights λ_t. The residuals e_i are assumed to be independent normally distributed with $E(e_i) = 0$ and $Var(e_i) = \sigma_e^2$.

The analysis consists of finding the regression weights λ_t, including the intercept λ_0, that optimally predict the developmental outcome. The developmental hypothesis at issue is evaluated by considering the reliability of the estimates of λ_t and more importantly, the percentage of explained variance in y_c. The root of the residual variance, that is σ_e, corresponds to the root mean square error of predictions. The developmental hypothesis is thought to be valid, when (some of) the predictors are significantly different from zero and the percentage of explained variance is reasonably large.

The standard regression model offers a very flexible statistical method to examine developmental hypotheses. The model can be extended easily with another set of developmental variables (e.g. repeated measurements of verbal reasoning together with repeated measurements of vocabulary size). In addition the criterion does not have to refer to the same construct as the predictors (e.g. when repeated measures of vocabulary are used to predict academic achievement). From a statistical point of view it is advantageous that the predictions are based on a small number of parameters relative to the number of observations.

There are, however, shortcomings too. Optimal prediction (maximum explained variance) is obtained when the predictors y_{ti} correlate highly with the response variable y_{ci}, and at same time low with each other. When predictors refer to the same *developmental* process the latter condition is often not met. The development vocabulary size offers a nice illustration of the reason why. A child's vocabulary at five years of age contains many words from his vocabulary at four years of age. His stock of words at four years of age contains many words of three years of age, and so on. This natural overlap results in substantial correlations between predictors. Thus, the conditions for optimal prediction by means of the regression model are not met.

Another drawback of the regression model is its disregard of the intervals between the measurement occasions. Each measurement corresponds to a specific age. The variable a_{it} (age) is however no part of the model. Finally, missing values need special attention. Missing values, either lead to list-wise deletion, or have to be replaced by means of imputation (Little & Rubin, 1987).

When used to test developmental hypotheses, the flexibility and statistical efficiency of the model do not compensate for its limitations. From a developmental perspective, correlations across occasions are very meaningful. They result from a common developmental process. Time, and thus the intervals between occasions, is part and parcel of this process (Wohlwill, 1973). Because the correlations and intervals between adjacent measurements are not taken into account properly, the standard regression model is not the best choice to test developmental hypotheses.

Dynamic Models

Another, intuitively more appealing way to test predictive developmental hypotheses is by means of dynamic models. In dynamic models the observations of person i are written as a function of age (or time). Testing developmental hypotheses takes two steps. First a function is fitted that describes the individual observations as a function of age or time: $y_{ti} = f(a_{ti}) + \xi_{ti}$. Next the fitted function is evaluated at $a = a_{T+1,}$ resulting in the predicted criterion $\hat{y}_{ci}$. Finally the extrapolated values $\hat{y}_{ci}$ are compared with the observed criterion y_{ci}.

The function f can take any form, depending on the developmental process considered. If little is known about the developmental process, as is often the case, one generally resorts to polynomials (Guire & Kowalski, 1979). The model is:

$$(2) \qquad y_{ti} = \eta_{0i} + \sum_{p=1}^{P} \eta_{pi} a_{ti}^{\,p} + \xi_{ti} \quad \text{with } \xi_{ti} \sim N(0, \sigma_{\xi i}^{2}).$$

It describes the developmental process for person i. y_{ti} and a_{ti} are respectively the *observation* and *age* of person i at occasion t (e.g. vocabulary size and age of a person during childhood). The observations are written as a P-degree polynomial function of age (a_{ti}). The parameters to be estimated are the intercept η_{0i} and the polynomial coefficients η_{pi}. The residuals ξ_{ti} are assumed to be independent normally distributed with $E(\xi_{ti}) = 0$ and $\mathrm{Var}(\xi_{ti}) = \sigma_{\xi i}^{2}$. Note that the parameters η_{pi} and the residual variance $\sigma_{\xi i}^{2}$ are specific to person i. No distributional assumptions are made about the parameters *across* persons. After the model has been fitted it is used to predict the criterion value y_{ci}. A natural option is to evaluate the fitted function at $a = a_{T+1}$.

To test the developmental hypothesis, the polynomials are estimated *individually* for each of the N persons in the sample. The subsequent evaluations of f at a_{T+1} result in the predicted values for criterion $\hat{y}_{ci}$. The error of prediction is $\varepsilon_i = y_{ci} - \hat{y}_{ci}$. The root mean square error of prediction is given by $RMSE = (\Sigma_i \varepsilon_i^2/N)^{1/2}$. The developmental hypothesis is thought to be valid if the percentage of explained variance of y_c is reasonably large. A significance test is performed through testing the correlation between the predicted and observed criterion against zero.

Testing the predictive hypothesis about the relationship between the development of vocabulary size during childhood and adult vocabulary would involve the following steps. First, for *each* individual a polynomial is fitted through his or her five childhood observations. Next a prediction of adult vocabulary size is obtained for each individual by substituting the adult age

($a_{T+1,i} = 21$) in his or her estimated polynomial equation. Finally a statistical comparison is made between the predicted and observed vocabulary size.

Although the given procedure appears to be attractive to the developmentalist it is rarely used. Its attractiveness stems from the fact that development is modeled at an individual level. Regrettably, the method is statistically deficient. The individually estimated polynomials are likely to be unreliable, because of the limited number of observations per person. In addition the fitted polynomials are even less reliable at the limits of and beyond the age range they span (Weisberg, 1985). Given these problems it is not surprising that the approach seldom leads to good models (Burchinal & Appelbaum, 1991)

Although the procedure has to be rejected for statistical reasons, its merits deserve mention. In contrast to the regression approach described previously, the model takes the intervals between occasions into account. In addition it allows for individual variation in growth trajectories. It handles missing values easily. For, if one of the early measurements is missing, a polynomial can still be fitted.

In sum the model is unreliable, but has some attractive features for developmentalists.

Hierarchical Linear Models

The longitudinal hierarchical linear model or multilevel model offers a better alternative to examine developmental hypotheses. Goldstein (1989) presented the model to be described in the context of physical growth (c.f. Goldstein, 1995). He used it to predict adult physical height. As we will show, the model is also well suited to test predictive developmental hypotheses.

Predictive developmental hypotheses consist of two parts, the early developmental process and the developmental outcome or criterion. The structure of the hierarchical linear model, to be described, closely reflects the structure of developmental hypotheses. The first part of the model describes the developmental behavior or process (e.g. the development of vocabulary size during childhood). The second part reflects the criterion or developmental outcome (e.g. adult vocabulary). Combining both parts results in a *bivariate* longitudinal hierarchical linear model.

In actual studies the developmental process is likely to be operationalized by means of repeated measurements of one or more variables. For reasons of clarity it is temporarily assumed that the developmental process of interest is charted by means of repeated measurements of a single variable. A more complex case will be considered later.

The longitudinal multilevel model (c.f. Bryk & Raudenbush, 1992, Chapter 6; Plewis, this issue) is used to describe the repeated measurements pertaining to the developmental process. It is a 2-level model, with repeated observations (level 1) nested within individuals (level 2).

The level 1 model describes the repeated observations of the developmental process as a P-degree polynomial function of age,

$$(3) \qquad y_{ti} = \pi_{0i} + \sum_{p=1}^{P} \pi_{pi} a_{ti}^{p} + e_{ti} \quad \text{with } e_{ti} \sim N(0,\sigma_e^2),$$

where y_{ti} and a_{ti} are respectively *observation* and *age* of individual i at occasion t; π_{0i} and π_{pi} (p = 1,2,..P) are the intercept and P polynomial coefficients of individual i. π_{1i} is designated the linear coefficient, π_{2i} is the quadratic coefficient, and so on. The so-called level 1 residuals e_{ti} are assumed to be independent normally distributed with $\mathrm{E}(e_{ti}) = 0$ and $\mathrm{Var}(e_{ti}) = \sigma_e^2$. More complex error structures, including serial correlation (Goldstein et al.,1994) and time dependent errors (Goldstein, 1995) can be accounted for, but will not be considered here.

The polynomial coefficients π_{pi} (including the intercept) are assumed to vary randomly across persons. The accompanying level 2 model for $p = 0,1,...,P$ is

$$(4) \qquad \pi_{pi} = \beta_p + r_{pi} \text{ with } r_{pi} \sim N(\mathbf{0},\mathbf{T}),$$

where π_{pi} is the $p^{\text{-th}}$ coefficient of individual i, β_p is the average p^{th} polynomial coefficient and r_{pi} the level 2 residual of person i. The residuals r_{pi} are assumed to be multivariate normally distributed with $E(r_{pi}) = 0$ and variance–covariance matrix **T.**

The full multilevel model describing the developmental process is obtained after the level 2 equation (Equation 4) is entered in the level 1 equation (Equation 3). The resulting equation after rearranging terms is:

$$(5) \qquad y_{ti} = \beta_0 + \sum_{p=1}^{P} \beta_p a_{ti}^{p} + r_{0i} + \sum_{p=1}^{P} r_{pi}\, a_{ti}^{p} + e_{ti}.$$

The first two terms on the right-hand side describe the average polynomial curve with the fixed coefficient β_0 to β_p. The next two terms reveal how the individual polynomials *deviate* from the average polynomial, by means of the random deviations r_{0i} to r_{pi}. The final term e_{ti} is the residual error.

The residuals r_{pi} have substantive meanings. The residual of the intercept r_{0i} corresponds to the status of the person at $a_{ti} = 0$; the residual of the linear coefficient r_{1i} refers to the developmental velocity and the quadratic residual r_{2i} pertains to 0.5 times the acceleration at that age. Together they characterize the individual developmental curves.

The model discussed so far only pertains to the developmental process (e.g. the development of vocabulary during childhood). We now turn to the developmental outcome or criterion to be predicted (e.g. adult vocabulary). The criterion consists of a single observation per person. As a result there is no level 1 variability. The model is

$$(6) \qquad y_{ci} = \beta_c + r_{ci} \text{ with } r_{ci} \sim N(0,\tau_c),$$

where y_{ci} is the observed value of the developmental outcome or criterion of person i at occasion $T + 1$. The first term on the right, β_c, is the mean developmental outcome. The second one, r_{ci}, is the deviation (residual) from the mean for person i. The residual r_{ci} has an obvious interpretation, it refers to the status of individual i relative to the other individuals.

In the final step the (uni-variate) model referring to the developmental process (Equation 5) and the (uni-variate) model pertinent to the outcome (Equation 6) are combined to a bivariate model. Two indicator variables are defined for that purpose. The first one is defined as follows, $\delta_{ti} = 1$ if y_{ti} refers to a measurement of the developmental process and $\delta_{ti} = 0$ otherwise. The second indicator is defined as $\delta_{ci} = 1$ if y_{ti} refers to the developmental outcome and $\delta_{ci} = 0$ otherwise.

The multivariate multilevel model combining the developmental part and the criterion is

$$(7) \quad y_{ti} = \delta_{ti}(\beta_0 + \sum\nolimits_{p=1}^{P} \beta_p a_{ti}^{p} + r_{0i} + \sum\nolimits_{p=1}^{P} r_{pi}\, a_{ti}^{p} + e_{ti}) + \delta_{ci}(\beta_c + r_{ci}),$$

where the variables and parameters have the same meaning as before, be it that y_{ti} refers to observations of the developmental process for $t = 1,..,T$ and to the developmental outcome for $t = T + 1$. The first part of Equation 7 between brackets refers to the developmental process (Equation 5). The second part between brackets refers to the developmental outcome (Equation 6). The level 1 residuals e_{ti} are again assumed to be independently normally distributed with $\mathrm{E}(e_{ti}) = 0$ and $\mathrm{Var}(e_{ti}) = \sigma_e^2$. It is further assumed that the level 1 residuals e_{ti} and the residual of the outcome r_{ci} are independent. It embodies the assumption that developmental outcome is only related to the early measurements through the developmental process, and not otherwise.

The level 2 residuals r_{0i} ... r_{Pi} and r_{ci} pertaining to the developmental process and developmental outcome are allowed to co-vary. The variance-covariance matrix of the level 2 residuals contains crucial information regarding the validity of the developmental hypothesis at issue. In many instances the order of the random coefficients of the developmental part does not exceed the second degree ($P = 2$). The variance-covariance matrix of the level 2 residuals pertaining to the developmental process (r_{0i} to r_{2i}) and the residuals of the criterion or developmental outcome (r_{ci}) is

$$(8) \qquad \mathbf{T}^* = \begin{pmatrix} \mathrm{var}(r_{0i}) & & & \\ \mathrm{cov}(r_{1i},r_{0i}) & \mathrm{var}(r_{1i}) & & \\ \mathrm{cov}(r_{2i},r_{0i}) & \mathrm{cov}(r_{2i},r_{1i}) & \mathrm{var}(r_{2i}) & \\ \mathrm{cov}(r_{ci},r_{0i}) & \mathrm{cov}(r_{ci},\mathrm{r}_{1i}) & \mathrm{cov}(r_{ci},r_{2i}) & \mathrm{var}(r_{ci}) \end{pmatrix}$$

The original matrix **T** of the level 2 residuals referring to the developmental process (Equation 4) is augmented with a row and corresponding column referring to the developmental outcome or criterion. The resulting matrix is $\mathbf{T}^*$.

The final row of $\mathbf{T}^*$ reveals the relationship between the residuals characterizing the developmental process, r_{0i} to r_{2i}, and the developmental outcome or criterion characterized by r_{ci}. To illustrate the meaning of the final row of $\mathbf{T}^*$ we return to our hypothetical example. We assume that the growth of vocabulary size during childhood follows a second-degree polynomial model with random coefficients up to the second degree ($P = 2$).

The final row of $\mathbf{T}^*$ is interpreted as follows. Cov(r_{ci},r_{0i}) indicates to what extent the developmental status at $a_{ti} = 0$ is related to the developmental outcome. When Cov(r_{ci},r_{0i}) is positive this would indicate that children with a large initial vocabulary will have a large vocabulary as an adult. Cov(r_{ci},r_{1i}) points to the relationship between the developmental velocity and the developmental outcome. A positive value of Cov(r_{ci},r_{1i}) indicates that a high growth rate during childhood results in a large adult vocabulary. Finally cov(r_{ci},r_{2i}) pertains to the relationship between the acceleration and the criterion or developmental outcome.

The final row (or column) of the variance-covariance matrix of the level 2 residuals already gives an indication of the validity of the developmental hypothesis at hand. Of course the covariance matrix can be transformed to a correlation matrix for ease of interpretation.

It should be stressed, that the estimates of the variances and covariances and thus the correlations depend on the scaling of the predictor a, measuring age or time (Schuster & von Eye, 1998). The estimates hold for $a = 0$.

Although the matrix already contains important information still another step has to be taken. The matrix is used to make *individual* predictions of the developmental outcome.

Predictions

The level 2 variance-covariance matrix of the multivariate hierarchical linear model describes the empirical relationship between the developmental process and the criterion. The next step is to use this matrix to *predict* the criterion observations.

Consider for a moment the possibility that the observations of a single new case arrive *after* the bivariate model has been estimated. In other words the following parameters are given, β_0 to β_P of the developmental process, β_c of the criterion, the level 2 variance covariance matrix $\mathbf{T}^*$ and the level 1 residual variance σ_e^2.

In the model the criterion is partitioned according to $y_{ci} = \beta_c + r_{ci}$. Given that the parameter β_c has been estimated, prediction of criterion y_{ci} for the new case comes down to prediction (or estimation) of the residual r_{ci}, from the observations y_{ti} of that case. Estimation or prediction of residuals is described by Goldstein (1995, Appendix 2.2).

To predict r_{ci} we first define the residualized observations $\tilde{y}_{ti} = y_{ti} - (\hat{\beta}_0 + \Sigma^P_{p=1} \hat{\beta}_p\, a_{ti}^{\,p})$. In addition we define $v^{-1}_{tt'}$ the elements of the inverse of the variance-covariance matrix $\mathbf{V}$ of the observations $\tilde{y}_{ti}$. Note that this matrix results from the estimates of $\mathbf{T}^*$ and σ_e^2 (Goldstein 1995, Appendix 2.1). The 'estimator' of the predicted residual is

$$\text{(9)} \qquad \hat{r}_{ci} = \sum\nolimits_{t=1}^{T} \sum\nolimits_{p=0}^{P} a_{ti}^{\,p} \operatorname{cov}(r_{ci}, r_{pi}) \sum\nolimits_{t'=1}^{T} \mathrm{v}^{-1}_{tt'}\, \tilde{y}_{t'i} \ ,$$

where $\hat{r}_{ci}$ is the predicted residual of the developmental outcome of person *i,* based on his or her (residualized) observations $\tilde{y}_{ti}$ made on occasions $t = 1,..T$ and P is the order of the level 2 random coefficients. The a_{ti} refer to ages of person i on occasions t. $\mathrm{Cov}(r_{ci}, r_{pi})$ is the level 2 covariance between the p^{th} random coefficient and the criterion.

Equation 9 is a so-called Empirical Bayes estimator. It combines two sources of information, data from the individual and data from the population (Snijders & Bosker, 1999). The first pertains to the individual's early observations ($\tilde{y}_{ti}$), the second to the estimates obtained from the model (especially the final row of $\mathbf{T}^*$ and the (co-)variances of the observations $\tilde{y}_{ti}$).

If no early data were available for a person, the a priori prediction of his or her developmental outcome would be $\hat{y}_{ci} = \hat{\beta}_c$ (i.e. the mean developmental outcome). But when one or more of his/her early observations becomes available the prediction is adjusted, and the a posteriori prediction becomes $\hat{y}_{ci} = \hat{\beta}_c + \hat{r}_{ci}$.

In sum predictions of the criterion ($\hat{y}_{ci}$) are made by computing Empirical Bayes estimates, based on the early developmental observations (y_{ti}).

Testing the Developmental Hypothesis

So far we discussed the bivariate longitudinal multilevel model linking the developmental process to the criterion, and the method for making individual predictions. To test a specific developmental hypothesis the following steps are taken. First a reliable model describing the relationship between the repeated measurements and the criterion (Equations 7-8) is sought. Significance of the fixed coefficients in the model is tested using their standard errors. The reliability of the random coefficients is evaluated by means of the likelihood ratio statistic (c.f. Goldstein, 1995). Once the model is found, the developmental outcome is predicted using Equation 9. The error of prediction is $\varepsilon_i = y_{ci} - \hat{y}_{ci}$. The root mean square error of prediction is given by $RMSE = (\Sigma_i \varepsilon_i^2/N)^{1/2}$.

The developmental hypothesis at issue is evaluated by considering the reliability of the estimated level 2 covariances describing the relationship between the developmental process and the criterion (the final row of $\mathbf{T}^*$, Equation 8) and by considering the percentage of explained variance in y_c. The developmental hypothesis is considered valid when the covariances with the developmental outcome are significantly different from zero and the percentage of explained variance is reasonably large.

The longitudinal multilevel model has a number of attractive features for developmental researchers. The structure of the model closely reflects the structure of the predictive developmental hypothesis. It accounts for the intervals between adjacent measurements in a proper way, whereas the parameters of the model are readily interpretable in terms of developmental status, velocity, and acceleration.

By means of Empirical Bayes estimates the model allows for reliable individual predictions. From a conceptual point of view it is important that the developmental variables and the developmental outcome do not have to refer to the same construct. From a practical point of view it is advantageous that the model handles missing values easily, both with respect to the developmental process and the developmental outcome. (If an outcome is missing the early observations still contribute to the precision of the growth parameters.) Last but not least the model can be extended with other sets of measurements as will be shown in the next section.

Extending the Model

Predictive developmental hypotheses are often not restricted to a single developmental process. Adult vocabulary size for instance may be related to the development of vocabulary size and to the development of verbal reasoning. To test the relationship between the combined developmental processes, the bivariate model has to be extended to a trivariate model.

We consider the following situation. Two developmental processes are observed. The observations of the first developmental process are designated by $y_{ti}^{(1)}$, where $t = 1,..,T^{(1)}$ refers to the measurement occasion and i to the person. The observations of the second developmental process are designated by $y_{ti}^{(2)}$, where $t = 1,..,T^{(2)}$ refer to the measurement occasion and i to the person. The criterion to be predicted is designated by y_{ci}.

The trivariate model is made of three components. The first two components describe developmental processes 1 and 2 by means of polynomial multilevel models

$$(10)\qquad \begin{aligned} y_{ti}^{(1)} &= \sum\nolimits_{p=0}^{P} \beta_p^{(1)} a_{ti}^p + \sum\nolimits_{p=0}^{P} r_{pi}^{(1)} a_{ti}^p + e_{ti}^{(1)}, \\ y_{ti}^{(2)} &= \sum\nolimits_{q=0}^{Q} \beta_q^{(2)} a_{ti}^q + \sum\nolimits_{q=0}^{Q} r_{qi}^{(2)} a_{ti}^q + e_{ti}^{(2)}. \end{aligned}$$

The Equations 10 parallel Equation 5. (Note that Equation 5 contained the intercept as a distinct term.) The variables $y_{ti}^{(1)}$ and $y_{ti}^{(2)}$ refer to the observations of developmental processes 1 and 2. The processes are described as an P-degree and Q-degree polynomial of age (a_{ti}). The coefficients $\beta_p^{(1)}$ and $\beta_q^{(2)}$ describe the average developmental curves of process 1 and 2. The random parameters $r_{pi}^{(1)}$ and $r_{qi}^{(2)}$ describe how the individual curves deviate from their respective average developmental curves (Their distribution will be discussed shortly). The residual errors $e_{ti}^{(1)}$ and $e_{ti}^{(2)}$ are assumed to be independently normally distributed with respectively $E[e_{ti}^{(1)}] = 0$, $\mathrm{Var}[e_{ti}^{(1)}] = \sigma_e^{(1)2}$, and $E[e_{ti}^{(2)}] = 0$, $\mathrm{Var}[e_{ti}^{(2)}] = \sigma_e^{(2)2}$. If necessary complex error structures can be accounted for (Goldstein et al.,1994; Goldstein, 1995).

The third component of the model is identical to Equation 6, it partitions the developmental outcome

$$(11)\qquad y_{ci} = \beta_c + r_{ci},$$

where y_{ci} the observed developmental outcome or criterion of person i, β_c the mean developmental outcome and r_{ci} the deviation from the mean (residual) of the observation of person i.

The next step is to combine the three components to one model, using indicator variables (c.f. 7). The combined model is

$$(12)\qquad \begin{aligned} y_{ti} = {} & \delta_{ti}^{(1)} [\sum\nolimits_{p=0}^{P} \beta_p^{(1)} a_{ti}^p + \sum\nolimits_{p=0}^{P} r_{pi}^{(1)} a_{ti}^p + e_{ti}^{(1)}] \\ & + \delta_{ti}^{(2)} [\sum\nolimits_{q=0}^{Q} \beta_q^{(2)} a_{ti}^q + \sum\nolimits_{q=0}^{Q} r_{qi}^{(2)} a_{ti}^q + e_{ti}^{(2)}] + \delta_{ci} (\beta_c + r_{ci}) \end{aligned}$$

The indicator variables are defined as follows. $\delta_{ti}^{(1)}=1$ if y_{ti} refers to a measurement of the developmental process 1 and $\delta_{ti}^{(1)}=0$ otherwise; $\delta_{ti}^{(2)}=1$ if y_{ti} refers to developmental process 2 and $\delta_{ti}^{(2)}=0$ otherwise. Finally δ_{ci} = 1 if y_{ti} refers to the developmental outcome and δ_{ci} = 0 otherwise.

The random parameters $r_{pi}^{(1)}$ and $r_{qi}^{(2)}$ describing the individual developmental curves, and the random parameter r_{ci} describing the developmental outcome are assumed to be multivariate normally distributed with $E[r_{pi}^{(1)}] = E[r_{qi}^{(2)}] = \mathrm{E}(r_{ci}) = 0$ and variance-covariance matrix $\mathbf{T}^*$ (c.f. Equation 8).

To illustrate $\mathbf{T}^*$ we assume that the model contains a random intercept and a random linear coefficient for both developmental processes. The level 2 covariance matrix $\mathbf{T}^*$ for $P = 1$ and $Q = 1$ is

$$(13)\ \mathbf{T}^* = \left\{\begin{array}{lllll} \mathrm{var}[r_{0i}^{(1)}] & & & & \\ \mathrm{cov}[r_{1i}^{(1)}, r_{0i}^{(1)}] & \mathrm{var}[r_{1i}^{(1)}] & & & \\ \mathrm{cov}[r_{0i}^{(2)}, r_{0i}^{(1)}] & \mathrm{cov}[r_{0i}^{(2)}, r_{1i}^{(1)}] & \mathrm{var}[r_{0i}^{(2)}] & & \\ \mathrm{cov}[r_{1i}^{(2)}, r_{0i}^{(1)}] & \mathrm{cov}[r_{1i}^{(2)}, r_{1i}^{(1)}] & \mathrm{cov}[r_{1i}^{(2)}, r_{0i}^{(2)}] & \mathrm{var}[r_{1i}^{(2)}] & \\ \mathrm{cov}[r_{ci}, r_{0i}^{(1)}] & \mathrm{cov}[r_{ci}, r_{1i}^{(1)}] & \mathrm{cov}[r_{ci}, r_{0i}^{(2)}] & \mathrm{cov}[r_{ci}, r_{1i}^{(2)}] & \mathrm{var}[r_{ci}] \end{array}\right\}$$

The first two rows of $\mathbf{T}^*$ correspond to the random level 2 parameters of developmental process 1. The next two rows, describe the covariances between the random coefficients of both developmental processes (first two columns) and the random parameters of developmental process 2 (next two columns). The final row contains important information with regard to the validity of the developmental hypothesis at issue. The random parameters refer to the relationship between the developmental level and developmental velocity of both processes with the criterion.

After the trivariate model (Equations 10-13) is fitted the parameter estimates are used to make individual predictions. First the observations of person i are residualized according to $\tilde{y}_{ti}^{(1)} = y_{ti}^{(1)} - \sum_{p=0}^{P}\hat{\beta}_p^{(1)} a_{ti}^p$ and $\tilde{y}_{ti}^{(2)} = y_{ti}^{(2)} - \sum_{q=0}^{Q}\hat{\beta}_q^{(2)} a_{ti}^q$. The Empirical Bayes estimator of the residual is now given by

$$(14)\ \hat{r}_{ci} = \sum_{t=1}^{T(1)+T(2)} \left\{\sum_{p=0}^{P} a_{ti}^{P}\mathrm{cov}[r_{ci}, r_{pi}^{(1)}] + \sum_{q=0}^{Q} a_{ti}^{q}\mathrm{cov}[r_{ci}, r_{qi}^{(2)}]\right\} \sum_{t'=1}^{T(1)+T(2)} v_{tt'}^{-1}\tilde{y}_{t'i}$$

The final step consists of adding the mean of the criterion $\hat{\beta}_c$ and the residual $\hat{r}_{ci}$ to arrive at $\hat{y}_{ci}$. How the trivariate and the previous models are applied will be shown in the next section.

Application

An application will be given involving the development of attachment behavior during infancy. Attachment behaviors are behaviors that promote proximity or contact with the caregiver. They are triggered by stressful events, for instance a short separation from the caregiver, a strange visitor at home, and illness (Bowlby, 1969). At a psychological level, attachment behaviors result in an increase of felt security.

The Strange Situation (Ainsworth et al., 1978) is a widely used standard laboratory procedure to induce and study attachment behavior. We will test the hypothesis that the child's attachment behavior in the Strange Situation at the beginning of the second year of life is the *outcome* of the *development* of the child's attachment behavior at home during the first year of life.

Sample and Design. The sample, consisting of 67 mother-child dyads, was taken from a longitudinal study on the development of mother-child interaction and attachment (Hoeksma & Koomen, 1991). The *development* of the attachment behavior was operationalized by means of two variables. First, the child's level of Contact Maintaining behavior was observed at home, at 3, 6, 9 and 12 months (details can be found in Hoeksma, Koomen & van den Boom, 1996). Second, mothers filled out the Perceived Attachment Behavior Scale (PABS) (Hoeksma & Koomen, 1991) when the children were 6, 9 and 12 months of age. The scale measures the level of attachment behaviors in everyday situations. The *outcome* (the criterion) was the child's level of Contact Maintaining behavior (Ainsworth et al., 1978) shortly after 12 months of age in the Strange Situation.

Data. The mean scores of Contact Maintaining and Perceived Attachment Behavior were $M = 1.87(SD = 1.57)$ and $M = 4.75(SD = 4.75)$, respectively. Figure 1 displays the raw data of the first 10 cases. It reveals that Contact Maintaining follows a rather irregular pattern in time, whereas the level of Perceived Attachment Behavior seems to increase in time for most individuals. The level of Contact Maintaining in the Strange Situation (the outcome to be predicted) was $M = 3.51$ ($SD = 1.98$). Data were analyzed using Ml*wiN* (Goldstein et al., 1998). Throughout the analyses Age was centered at 9 months.

Preliminary Analyses. In the context of multilevel modeling it is wise to start with simple models first. We started with univariate longitudinal models (Equations 3-5), to get an impression of the developmental course of the variables involved.

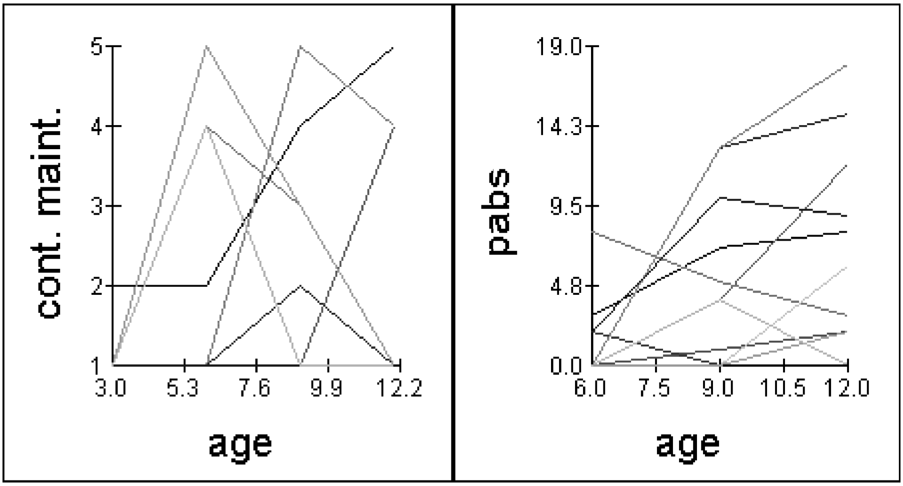

Figure 1
Raw Data of Ten Cases

During the analyses, fixed coefficients were added when they exceeded twice their standard error. Significance of added random terms is tested by means of the likelihood ratio test (Goldstein 1995). Results of the preliminary analysis have been reported earlier in Hoeksma and Koomen (1991), and Hoeksma et. al, (1996).

The analysis of both variables resulted in a linear model with a random intercept and a random linear coefficient. Table 1 (Column 2 and 3) contains the parameter estimates. Note that the linear coefficient of Contact Maintaining does not exceed twice its standard error [$\beta_{1(\mathrm{Cont.Maint.})} = .003$, $SE = .033$]. It was nevertheless retained in the model to get a proper estimate of its level 2 variance. Figure 2 portrays the individual developmental curves based on the estimated models. The developmental curves of Contact Maintaining appear to cross each other, whereas the curves of Perceived Attachment Behavior spread out with increasing age.

Bivariate Model. Next a bivariate model (Equation 7) was evaluated, to test the hypothesis that the *development* of Contact Maintaining at home affects the *level* of Contact Maintaining in the Strange Situation.

The model describing the development of Contact Maintaining was extended to a bivariate model by adding observations of Contact Maintaining in the Strange Situation to the original response variable. In addition two dummy variables were created (δ_{ti} and δ_{ci}, Equation 7) to indicate to which part of the model the observations and corresponding predictors belong.

Table 1

Longitudinal Multilevel Models of Contact Maintaining and Perceived Attachment Behavior (PABS), and the Prediction of Contact Maintaining in the Strange Situation: Parameter Estimates and (Standard Errors)

	Contact Maintaining	PABS	Bivariate Model	Trivariate Model
Fixed:				
$\beta_{0(\text{Cont. Maint})}$	1.88 (.11)		1.88 (.11)	1.88(.11)
$\beta_{1(\text{Cont. Maint})}$	.003(.033)		.003(.033)	.003(.033)
$\beta_{0(\text{PABS})}$		5.13(.50)		4.75(.44)
$\beta_{1(\text{PABS})}$		.76 (.10)		.76(.10)
$\beta_{0(\text{Attachment})}$			3.51 (.24)	3.51 (.24)
Random (Level 2):				
$\text{Var}[r_{0i(\text{Cont.Maint})}]$	.24 (.16)		.24 (.16)	.24(.16)
$\text{Var}[r_{1i(\text{Cont.Maint})}]$	.031(.014)		.031 (.014)	.031(.14)
$\text{Cov}[r_{01i(\text{Cont.Maint})}]$	.062(.035)		.062 (.035)	.062 (.35)
$\text{Var}[r_{0i(\text{PABS})}]$		11.04(2.25)		10.69 (2.25)
$\text{Var}[r_{1i(\text{PABS})}]$		.38(.13)		.37(.13)
$\text{Cov}[r_{01i(\text{PABS})}]$		.88(.38)		.88(.38)
$\text{Cov}[r_{0i(\text{Cont.Maint})}, r_{0i(\text{PABS})}]$				1.59(.45)
$\text{Cov}[r_{1i(\text{Cont.Maint.})}, r_{0i(\text{PABS})}]$				.26 (.12)
$\text{Cov}[\text{r}_{0i(\text{Cont.Maint.})}, r_{1i(\text{PABS})}]$				.17(.096)
$\text{Cov}[r_{1i(\text{Cont.Maint.})}, r_{1i(\text{PABS})}]$				.007(.028)
$\text{Var}(r_{ci})$			3.87(.67)	3.87(.67)
$\text{Cov}[r_{ci}, r_{0i(\text{Cont.Maint.})}]$			.79(.24)	.79(.24)
$\text{Cov}[r_{ci}, r_{1i(\text{Cont.Maint.})}]$			.16(.068)	.16(.068)
$\text{Cov}[r_{ci}, r_{0i(\text{PABS})}]$				2.86(.93)
$\text{Cov}[r_{ci}, r_{1i(\text{PABS})}]$				.46(.21)
Residual (Level 1):				
$\sigma_{e\ (\text{Cont.Maint}).}^{2}$	1.99 (.24)		1.99 (.24)	1.99(.24)
$\sigma_{e\ (\text{PABS}).}^{2}$		5.50(.96)		5.72(.96)

Note: Columns 2 and 3: Univariate longitudinal models for Contact Maintaining and Perceived Attachment behavior. Column 4 and 5: Prediction of Contact Maintaining in the Strange Situation. Age centered at 9 months.

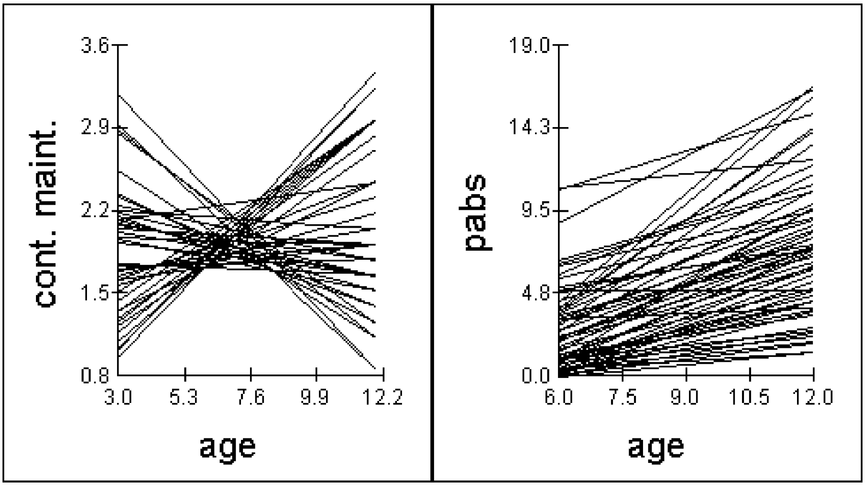

Figure 2
Developmental Curves

Parameter estimates of the resulting *bivariate* model are displayed in Table 1 (Column 4). The estimates of the fixed parameters $\beta_{0(\text{Cont.Maint.})}$ and $\beta_{1(\text{Cont.Maint.})}$ describing the average developmental curve of Contact maintaining at home, are close to the estimates of the univariate model. The same holds for the random parameters $\text{Var}[r_{0i(\text{Cont.Maint})}]$, $\text{Var}[r_{1i(\text{Cont.Maint})}]$ and $\text{Cov}[r_{01i(\text{Cont.Maint})}]$. If these random parameters were used to portray the individual developmental curves of Contact Maintaining at home, this would lead to a similar picture as in Figure 2.

The level 2 parameters $\text{Cov}[r_{ci}, r_{0i(\text{Cont.Maint.})}]$ and $\text{Cov}[r_{ci}, r_{1i(\text{Cont.Maint.})}]$ pertain to the relation between the *development* of Contact Maintaining and the level of the same behavior in the Strange Situation. Because their estimates are close to the estimates of the next model to be discussed we postpone their interpretation to the next section.

The last step of the analysis involved the prediction of the level of Contact Maintaining in the Strange Situation (see Equation 9). The correlation between the observed and predicted level of Contact Maintaining was $r = .45$ ($p <. 05$, $N = 67$) corresponding to 20% of the variance. The root mean square error of prediction was $RMSE = 1.77$. The results so far indicate that Contact Maintaining in the Strange Situation is related to the *development* of Contact Maintaining at home.

Trivariate Model. It was hypothesized that Contact Maintaining in the Strange Situation (the developmental outcome or criterion) was not only affected by the development of Contact Maintaining at home, but also by the development of Perceived Attachment Behavior. In the final model evaluated, both variables were used to predict the behavior in the Strange Situation.

The bivariate model describing the relationship between Contact Maintaining at home and Contact Maintaining in the Strange Situation was extended to a trivariate model. The repeated observations of Perceived Attachment Behavior were added to the response variable of the previous bivariate model. Besides the indicator variable $\delta_{ti(\text{Cont. Maint.})}$ indicating observations of Contact Maintaining at home and the indicator δ_{ci} pointing to observations of Contact Maintaining in the Strange Situation, a third indicator was created $\delta_{ti(\text{PABS.})}$ to indicate the presence/absence of the observations corresponding predictors of Perceived Attachment Behavior.

Table 1 (column 5) displays the parameter estimates of the fitted model. The estimates pertaining to the developmental behaviors at home appeared to match the estimates of the previous analyses again. In other words the multivariate model leads to the same interpretations with regard to the development of Contact Maintaining and Perceived Attachment Behavior as the univariate models. As Figure 2 already revealed, the individual developmental curves of Contact Maintaining cross each other, whereas the curves of Perceived Attachment Behavior spread out with increasing age.

The random parameters $\text{Cov}[r_{ci},r_{0i(\text{Cont.Maint.})}]$ to $\text{Cov}[r_{ci},r_{1i(\text{PABS})}]$in Table 1 (Column 5) reveal the relationship between the *development* of Contact Maintaining and Perceived Attachment Behavior (PABS) on the one hand, and the *level* of Contact Maintaining in the Strange Situation on the other.

Table 2 displays the relevant estimates once more. For ease of interpretation the covariances are converted to correlations. There appeared to be substantial correlations of the random intercepts (levels) of both Contact Maintaining (r = .82) and Perceived Attachment Behavior (r = .44) with the level of Contact Maintaining in the Strange Situation. Note that the correlations hold for the levels at 9 months of age, because the age variable was centered at 9 months.

Substantial correlations were also observed for the linear coefficients of Contact Maintaining (r = .45) and Perceived Attachment Behavior (r = .38) with the behavior in the Strange Situation. The correlations are valid for the full age range considered, because the developmental curves are linear (see Figure 2). They suggest that the *rate of change* of both developmental variables affects the behavior in the Strange Situation.

The last step of the analysis involved the prediction of the level of Contact Maintaining in the Strange Situation on the basis of the parameters of the model (c.f. Equation 14). The correlation between the observed and

Table 2
Relationship between the Development of Contact Maintaining and Perceived Attachment Behavior (PABS) at Home and the Level of Contact Maintaining in the Strange Situation: Level 2 Covariances (including Standard Errors) and Correlations

		Contact Maintaining		PABS	
		(r_{ci}, r_{0i})	(r_{ci}, r_{1i})	(r_{ci}, r_{0i})	(r_{ci}, r_{1i})
Strange	Cov	.79(.24)	.16(.07)[a]	2.86(.09)	.55(.21)[b]
Situation	r	.82	.45	.44	.38

Note. Age centered at 9 months.
[a]($G^2 = 5.84$, $df = 1$, $p < 0.01$). [b]($G^2 = 5.30$, $df = 1$, $p < 0.01$).

predicted level of Contact Maintaining was $r = .52$ ($p < .01, N = 67$), accounting for 27% of the variance. The corresponding root mean square error of prediction was $RMSE = 1.70$.

Conclusion. The relation between the development of Contact Maintaining and Perceived Attachment behavior was analyzed in three steps. The univariate model revealed clear individual differences with respect to the *development* of both attachment behaviors. The next analysis showed that the development of Contact Maintaining at home accounts for 20% of the variance of Contact Maintaining in the Strange Situation. Adding the development of Perceived Attachment behavior accounted for an additional 7% of the variance. Not only the *level* of attachment behavior, but also *changes* in attachment behavior, appeared to be related to the behavior in the Strange Situation after the infant's first year. These results clearly confirm the hypothesis that the child's attachment behavior in the Strange Situation is related to the *development* of attachment behavior at home during the first year of life.

Discussion

Predictive developmental hypotheses play a crucial role in developmental theories. Hypotheses that allow for individual predictions are preferred above hypotheses leading to predictions about means. This makes regression analysis the natural choice to investigate predictive developmental hypotheses. We argued that the model is less suited to test predictive developmental hypotheses because it disregards the special characteristics of developmental data. Extrapolation of individual growth curves appeared more attractive but is often unreliable.

In our view the longitudinal multilevel model combines the best of both (rejected) approaches. It is flexible and statistically efficient in the same way as the regression model. A relative small number of parameters is estimated given the number of observations. Its intuitive attractiveness matches the attractiveness of the individually estimated dynamic model. The model allows for individual predictions from individual developmental data.

Growth and development can also be modeled within the framework of Structural Equation Modeling (Muthén 1991, Willett & Sayer, 1994). The same holds for the longitudinal model presented here. In our view there is no principle reason to prefer one model above the other. Both conceptual and practical reasons should guide one's choice.

The multilevel model was described for, and applied to interval measurements. It is a small step to replace the interval measurements of the criterion by nominal observations or count data and their appropriate distributions (Goldstein, 1995). A further extension is to measure the criterion more than once and to accommodate the model accordingly.

One of the main virtues of the model presented is that its structure closely reflects the structure of predictive developmental hypotheses. The first part of the model describes the variability of individual developmental histories or trajectories. The second part describes various outcomes of these histories. The full model combines the characteristics of the histories or trajectories with the outcomes they are thought to bring about. Developmental histories or trajectories are characterized by *change*. The predictive hierarchical linear model acknowledges developmental changes.

The predictive hierarchical linear model is likely to have theoretical implications. Because the model takes developmental changes explicitly into account, it forces developmental psychologists to think about change and not only about the results of change.

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